A.N.Shelaev

OVER LONG CHANGE OF LOW-STABLE SELF-MODULATION REGIMES BY HIGHLY STABLE STATIONARY REGIMES WITH VERY LONG MEMORY OF INDUCED REGIMES IN $YAG:Nd^{3+}$ SOLID-STATE RING LASERS AT HIGH-FREQUENCY MODULATION OF LASER PERIMETER WITH EXTREMELY SMALL AMPLITUDE MUCH LESS RADIATION WAVELENGTH

D.V.Scobeltsyn Institute of Nuclear Physics (SINP)
Lomonosov Moscow State University
Shelaev@mail.ru

Abstract

It is reported about experimental establishment and theoretical interpretation of nontrivial effects of abnormally long change $(\geq 10-100\,\mathrm{s}\ !!)$ of the low-stable 1st and 2nd kind self-modulation regimes by exclusively stable stationary regimes of uni- or bidirectional lasing with constant intensities and effects of very long $(\geq 10\,\mathrm{s}\,!)$ memory of this induced regimes after break of lasing in $YAG:Nd^{3+}$ solid-state ring lasers (SRL) under modulation of laser perimeter at high-frequencies $(\geq 1\,\mathrm{MHz})$, which corresponds to return time of setting of the field in the laser resonator, and at extremely small perimeter modulation amplitude $(\leq 10^{-8}-10^{-9}\,\mathrm{m}\,!)$, which is much less then the radiation wavelength ($\lambda \simeq 1{,}064\cdot 10^{-6}$m).

The solid-state ring lasers (SRL) on $YAG:Nd^{3+}$- crystal active medium (AM) with the homogeneously broadened gain line and a long time of a relaxation of inverted population $T_1 \simeq 2{,}3\cdot 10^{-4}s$, which is much greater than the time of setting of the field $t_C \simeq Q/\eta\omega \leq 10^{-7}s$ ($\omega = 2\pi/\lambda$ - radiation frequency, Q - good quality of the laser resonator, $\eta=(W-W_{th})/W_{th}$ - excess of threshold power W_{th} of pumping) are unique object for experimental and theoretical studying of very unusual effects of nonlinear dynamics [1-3].

The matter, firstly, is that in SRL with crystal AM unlike in gas RL takes place the strong competition between the light counterrunning waves (CRW) as in such SRL CRW interact with the same motionless atoms of the AM. While in gas RL under detuning generation frequency from the center of the gain line CRW owing to Doppler's effect interact with various moving atoms. Secondly, in SRL along with usual linear coupling of the CRW due to the backward scattering on the ununiformity of dielectric permeability ε exists an inertial nonlinear coupling of the CRW in the AM due to the backward scattering on the induced by CRW structures of inverted population. In gas RL owing to the movement of atoms and small time of $T_1 \sim 10^{-7}\ s$, the induced structures aren't created and nonlinear coupling on them is absent. At last, thirdly, in the SRL AM has large amplifying coefficient due to high concentration of active atoms and, therefore, in the SRL nonlinear effects are shown much stronger, than in the gas RL.

It is established experimentally that in the nonrotating SRL in the absence of modulation of the SRL perimeter depending on the parameters of coupling of the CRW, determined by phenomenological complex coupling coefficients $\mathrm{m}_{1,2}=m_{1,2}\cdot\exp(\pm i\theta_{1,2})$ ($m_{1,2}=(c/L)\sqrt{r_{1,2}}$, c - velocity of light, L - resonator perimeter, $r_{1,2}$ - coefficients of the linear power backward scattering), the level of pumping η, number of generating modes and the detuning $\delta=(\omega-\omega_0)/\Delta\omega_l$ of generation frequency ω from the center of the amplifying line ω_o ($\Delta\omega_l \simeq 196$ GHz - half-width of the amplifying line) the following generation regimes are realized:

1) the multimode regimes of a standing wave with constant and equal intensities of the CRW, which are observed at strong coupling ($r_{1,2} \geq 10^{-2}$); 2) the single-mode regimes of the running wave (the unidirectional generation), existing at weak

coupling of the CRW (end faces of the AM are cut off at a Brewster angle or aren't perpendicular to laser beam and are brightened up, $r_{1,2} \leq 10^{-3}$); 3) the regimes of self-modulation of the 1st kind (see fig. 1) in the simplest case with antiphase sinusoidal modulation of the CRW intensities at frequencies $\omega_M \simeq \sqrt{m_1 \cdot m_2} \simeq (10^2 - 10^6)$ Hz, which are realized at weak coupling of the CRW, and, as a rule, in the presence in the SRL a selector of longitudinal modes; 4) the 2nd kind self-modulation regimes (see fig. 2) with spontaneous quasi-periodic $\omega_M \simeq (10^{-1} - 10^3)$ Hz change of the direction of radiation, happening after attenuation of the transition process at a relaxation frequency $\omega_r = \sqrt{\eta\omega / QT_1} \geq (10 - 100)$ kHz at weak coupling of the CRW and at detuning of the generation frequency from the center of the gain line greater than critical detuning $\delta_{cr} \approx (1+\eta)\sqrt{\eta\omega T_1 / Q}$.

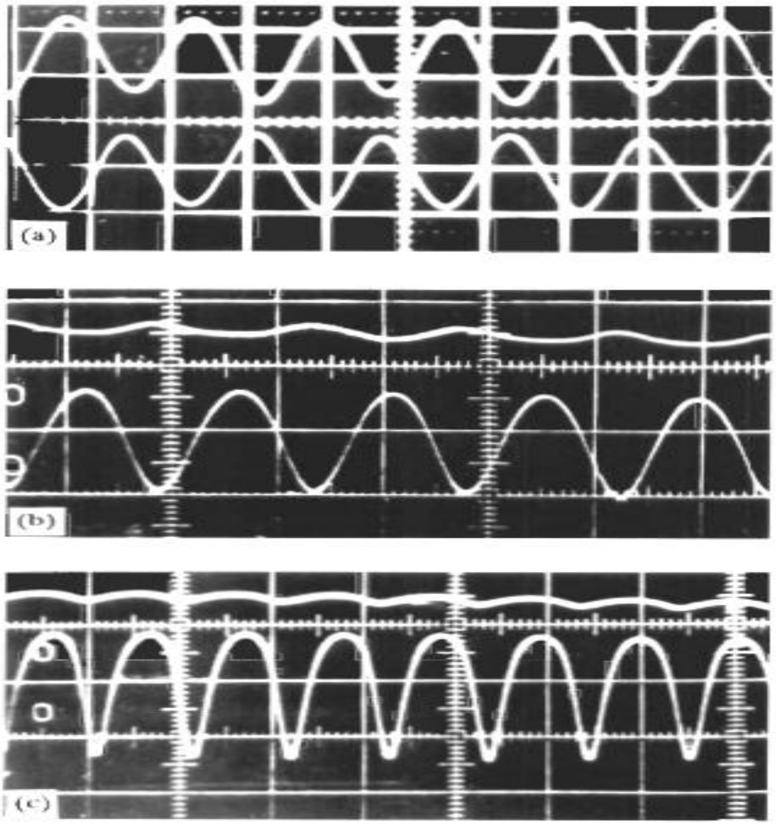

Fig1. Oscillograms of the CRW intensities in the 1-st kind self-modulation regimes with antiphase modulation and equal or unequal intensities, with inphase modulation and unequal intensities. The scan scale is 50 $\mu s / div$.

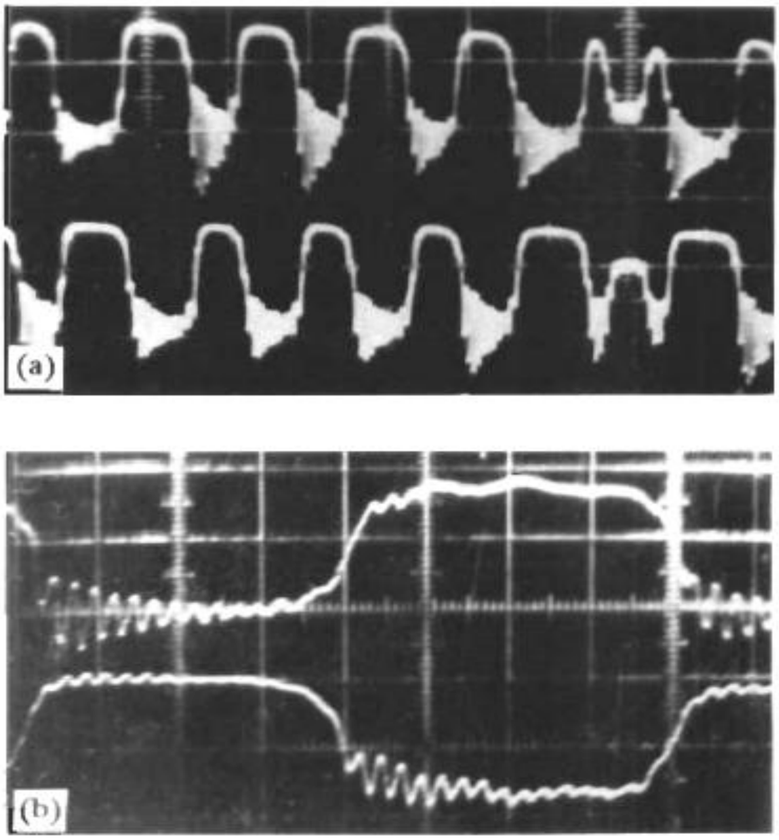

Fig. 2. Oscillograms of the CRW intensities in the 2-nd kind self modulation regimes. Scan scale is $1ms/div$ (upper oscillogram) and $200\,\mu s/div$ (lower oscillogram).

In theoretical investigations the light field **E** in the RL resonator is written as the sum of two CRW propagating along the resonator z-axis:

$$\mathbf{E} = \mathrm{Re}\{\sum_{1,2} \vec{e}\mathrm{E}_{1,2}(\mathrm{t}) \cdot \exp(\mathrm{i}(\omega \mathrm{t} \pm \mathrm{kz})\} \qquad (1),$$

where $\mathrm{E}_{1,2}(\mathrm{t}) = E_{1,2} \exp(i\varphi_{1,2})$; $\mathrm{E}_{1,2}(\mathrm{t})$, $E_{1,2}, \varphi_{1,2}$ - are the complex amplitudes, the modulus values and the phases of the CRW fields respectively; $\vec{e}$ - is the unit polarization vector.

Assuming that the amplitudes $\mathrm{E}_{1,2}(\mathrm{t})$ are slowly varying, it can be derived from the Maxwell equations the reduced equations for the complex amplitudes:

$$\frac{d\mathrm{E}_{1,2}}{dt} = -\mathrm{E}_{1,2}\omega / 2\mathrm{Q}_{1,2} \pm \mathrm{E}_{1,2}\frac{\mathrm{i}\Omega}{2} + \mathrm{E}_{2,1}\frac{\mathrm{i}m_{1,2}}{2} + (1 - \mathrm{i}\delta)\cdot(\mathrm{N}_0\mathrm{E}_{1,2} + \mathrm{N}_{\mp}\mathrm{E}_{2,1})\frac{\sigma d}{2T} \qquad (2),$$

here the terms $\pm i\Omega E_{1,2}/2$ gorven phase (frequency) nonreciprocity of the ring resonator for the CRW, Ω - is the difference of eigen-frequencies $\omega_{1,2}$ of ring resonator for the CRW ($\Omega=\omega_1-\omega_2$) due to, for example, Sagnac effect under rotation of the SRL ($\Omega/2\pi=8\pi Sf_R/\lambda L$, S - the area of the ring resonator, f_R - the angular velocity of the RL rotation); $T=L/c$ - is the ring resonator round–trip time, $\sigma=\sigma_0/(1+\delta^2)$ - is the laser transition cross section.

$N_0=(1/d)\int_0^d N dz$ and $N_\pm=(1/d)\int_0^d N\exp(\pm i2kz)dz$ are zero and second spatial garmonics (with $\lambda/2$ period) of the inverted population N in the AM (d - is active element length), which are defined by the equations:

$$T_1\frac{dN_0}{dt}=N_{th}(1+\eta)-N_0[1+a(|E_1|^2+|E_2|^2)]-aN_+E_1E_2^*-aN_-E_1^* \qquad (3),$$

$$T_1\frac{dN_\pm}{dt}=-N_\pm[1+a(|E_1|^2+|E_2|^2)]-aN_0E_1^*E_2 \qquad (4)$$

So, the initial set of equations (2-4) governing the dynamics of SRL is integro-differential. Due to interference of the CRW in the SRL AM with $T_1 \gg t_c \simeq Q/\eta\omega$ inertial periodic structures (gratings) are induced in the AM. These gratings are similar to the gratings, produced in hologram recording. The self-diffraction of the CRW on the inverted population gratings is responsible for the nonlinear coupling of CRW in the AM. This coupling is described by terms $N_\pm E_{1,2}$ in (2).

If $\delta \ll 1$ the gain coefficients of the CRW can be written as follows

$$\kappa_{1,2}=(N_0+N_\pm E_{2,1}/E_{1,2})\sigma d/2=\kappa_0(1-\alpha a_S E_{1,2}^2-\beta a_S E_{2,1}^2) \qquad (5),$$

where κ_0 - is the small signal $(\eta \ll 1)$ gain coefficient, α, β are the auto- and cross-saturation coefficients ($\alpha=1$, $\beta=1+1/(1+(\Delta\omega T_1)^2)$, a_S - saturation parameter), $\Delta\omega$ is the difference of the CRW frequencies.

And therefore the difference between the gain coefficients for the CRW is given by

$$\kappa_1-\kappa_2=(\beta-\alpha)a_S(E_1^2-E_2^2)\kappa_0 \qquad (6)$$

As $\beta > \alpha$ the wave with higher intensity has a larger gain coefficient. Due to this competitive suppression of one of the CRW takes place in the SRL. Thus in the absence of linear coupling of the CRW via backward scattering $(m_{1,2} = 0)$ in the SRL exists unidirectional generation for frequency detunings

$$|\delta| < \delta_{cr} = (1+\eta) / \sqrt{T_1 \eta \omega / Q} \qquad (7)$$

By increasing the backward scattering the situation can be changed and in this case the CRW with higher intensity has a lower effective gain coefficient when

$$m_1 \simeq m_2 = m \quad \text{and} \quad m \cdot |\sin[(\theta_1 - \theta_2)/2]| > \eta\omega/Q \qquad (8)$$

As it follows from (8), the backward scattering stabilizes the bidirectional generation as much as possible in the case of anti-complex conjugated coupling coefficients ($|\theta_1 - \theta_2| = \pi$).

At $\delta > \delta_{cr}$ and $m_{1,2} \approx 0$ in the nonrotating ($\Omega = 0$) SRL it is unstable at the same time both the unidirectional, and bidirectional generation.

One of the possibilities for gorvening SRL radiation dynamics is the use of nonstationary resonator [1]. When the SRL resonator perimeter is periodically modulated due to vibrations of one of the ring resonator mirrors, the SRL is found to feature a number of intriguing effects associated with averaging of the CRW coupling and inverted population gratins induced in the AM.

New lasing regimes and competing effects of interaction between the CRW were observed under conditions of low-frequency (*f*~100 Hz) vibrations of one of the ring resonator mirrors. For elimination of misalinment of the ring resonator at vibrations of mirror the configuration of the SRL was such, that the angle of incidence of the light CRW on this mirror was small $\sim 1^o$. As a result the following regimes were observed in the SRL: regimes with cophase or antiphase modulation of the CRW intensities, regimes with different frequencies and different modulation of the CRW intencities (see fig.3), regimes with variable-frequency antiphase oscillations of the CRW intensities, regimes with trains of cophase pulses and very interesting for applications regimes of kinematic mode-locking, which take place at vibtational amplitudes $a \geq 10\lambda$. It is necessary to notice that at change the RL perimeter on λ the generation frequency changes on value, which is equal to the frequency of intermode beats c/L.

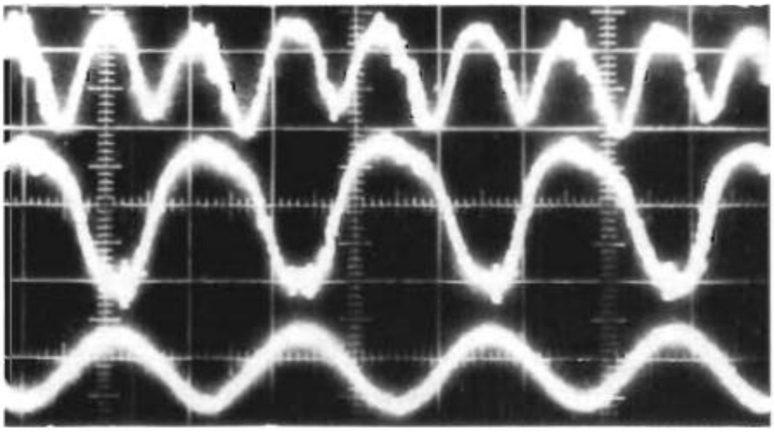

Fig. 3. Oscillogram of the CRW intensities (two upper beams) and mirrow vibrations (low beam) in the regime with unequal frequencies and amplitudes of modulation of the CRW. The mirror vibrational amplitude $a \simeq \lambda$, the frequency $f \simeq 210\, Hz$.

In this article the emphasis is placed on the most intriguing effects in nonlinear dynamics of the SRL at over small amplitudes of mirror vibrations $a \sim 10^{-2} - 10^{-3}\,\lambda$ on frequencies corresponding to the return time of establishment of the field in the laser resonator $f \sim 10^{6}\, Hz$.

It has been established experimentally that even at such insignificant amplitudes ($a \sim 10^{-9}\, m$) of modulation of the resonator perimeter in the nonrotating $YAG:Nd^{3+}$ SRL occurs, firstly, abnormally long (during times $\Delta t \sim 10 - 100\, s$, which are about 5 (!) orders more than the longest laser parameter $T_1 \simeq 2{,}3 \cdot 10^{-4}\, s$ - the time of a relaxation of inverted population) change of the low-stable self-modulation regimes of the 1st and 2nd kind by very stable uni- or bidirectional generation with constant intensities of the CRW (see fig. 4, 5).

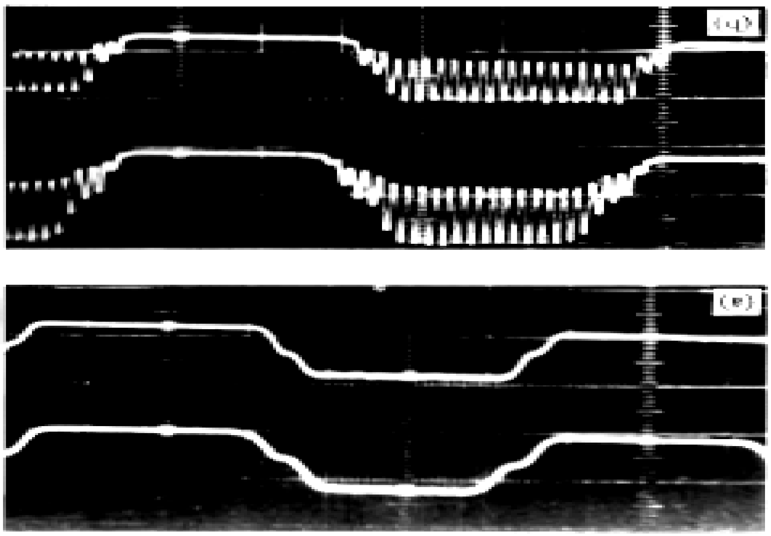

Fig. 4. Oscillograms of the CRW intensities in the SRL with immobile mirror in the 1-st kind self-modulation regime (upper oscillogram) and in the same SRL with vibrating mirror in stationary bidirectional regime with constant intensities of the CRW (lower oscilliogram). Vibrational frequency and amplitude of the mirror $f = 1450\,Hz,\ a \sim 10^{-2}\,\lambda \simeq 10^{-6}\,m$. Time of change of the self-modulation regime by stationary regime $\Delta t \sim 10\,s$. Scan scale is $200\,\mu s / div$.

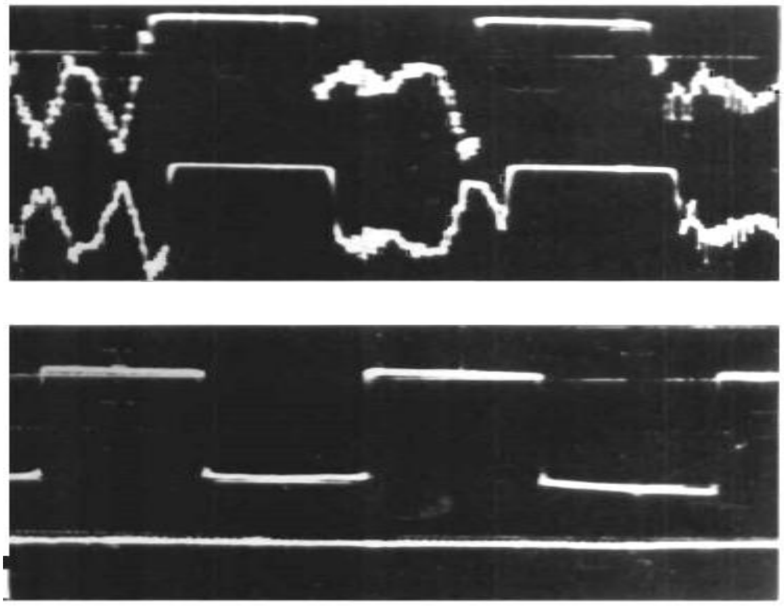

Fig. 5. Oscillograms of the CRW intensities in the SRL with immobile mirror in the 2st kind self-modulation regime (upper oscillogram) and in the same SRL with vibrating mirror in stationary bidirectional regime with constant intensities of the

CRW (lower oscilliogram). Vibrational frequency and amplitude of the mirror $f = 1450\,Hz$, $a \sim 10^{-3}\lambda \simeq 10^{-9}\,m$. Time of change of the self-modulation regime by stationary regime $\Delta t \sim 10\,s$. Scan scale is $1\,ms/div$.

For display of a constant component of radiation the CRW intensities in fig. 4,5 are modulated by the rotating breaker (chopper) placed before photodetectors.

Secondly, in the SRL with non-stationary resonator it has been found also the effect of superlong ($\Delta t \geq 10\,s$) memory of the induced generation regimes which is shown at upsetting of generation when placing in the SRL resonator the opaque screen. If times of upsetting of generation were less than $\leq 10\,s$, then after removal of the screen in the SRL the induced regimes with constant intensities of the CRW were restored practically at once. At times of upsetting of generation of $\Delta t \geq 10\,s$ in the SRL with the non-stationary resonator the initial self-modulation regimes were restored and than again very slowly passed into the regimes with constant intensities of the CRW.

In the third, at the end of the process of setting of the induced stationary regimes it takes place the characteristic, as if farewell, convulsive splash of the CRW intensities. In this case the self-modulation regime is similar to the animated being leaving us.

In the fourth, if in the nonrotating SRL with the non-stationary resonator the induced regime of bidirectional generation was set, then, after creation of a difference of frequencies of the CRW bigger than the bandwidth of capture of the frequencies of the CRW ($\Omega > \Omega_0$), in the rotating SRL the regime of beats, much stable than in absence of vibrations of a mirror, was realized.

Numerical calculations on the basis of system of the equations (2-4) have also shown very slow reduction of amplitude of self-modulation at very small ($a \ll \lambda$) high-frequency ($f \sim \eta\omega/Q$) modulation of the SRL perimeter.

The question of physical mechanisms of long-term memory of the induced regimes after break of generation, found by us experimentally in the SRL with nonstationary resonator - very complex nonlinear system, described by complicated integro-differential system of the equations (2-4), remains still open. In this regard we will point to the following circumstances.

Experimental studies of the $YAG:Nd^{3+}$ SRL and numerical calculations of initial system of the equations (2-4) have allowed us to establish existence the 2nd kind self-modulation regimes with ultralow periodic ($f < 10^{-1}\,Hz$) change of the direction of radiation.

The physical reasons for the found ultralow switching are nontrivial and aren't reduced to effect of long gradual transfer of energy between weakly connected generators with usual linear coupling. It has been established that the effect of ultralow switching takes place at large detunings the generation frequency from the center of the gain line ($\delta > \delta_{cr}$) and is caused, in particular, emergence even in the nonrotating (!) SRL the CRW frequency splitting, which is periodically changing in time and is equal to the relaxation frequency, characteristic for the SRL $\omega_{r1} == \omega_r / \sqrt{2}$. The difference of frequencies of the CRW leads to the oscillation of spatial harmonics of inverted population and to nonlinear coupling of the CRW that, in turn, leads to change of signs of decrements of attenuation of the CRW and to very slow development of non stabilities (bifurcations) Andronov-Hopf’s type.

It should be noted that the effect of long-term memory has been found by us also at numerical calculations of the famous Lorenz model, which consists of “simple” system of ordinary differential equations [4,5]. This model describes Rayleigh-Binara's convection at warming up from below a layer of liquid of final thickness, so that between the lower and top surfaces the constant difference of temperatures is supported, and consists of system of 3 ordinary differential equations for variables X, Y, Z:

$$dX/dt = -\sigma X + \sigma Y, \quad dY/dt = rX - Y - XZ, \quad dZ/dt = XY - bZ \tag{9}$$

Gradually changing the values of numerical parameters b, r, σ of this system at restarts of the program of the account it was succeeded to reach the forbidden values of parameters at which the account didn't begin at the first start of the program.

That is, in the course of the repeated account this system - the computer plus the mathematical program - as if adapted on new parameters. As a result, very unusual effects of splitting of a limit cycle and structuring of phase trajectories have been gained up to their full dispersal “in dust”. It was possible to erase memory of the results of previous account only at reset of the computer.

In this regard it is worth emphasizing that the specialists working with neural networks still can't distinctly explain the processes of self-training of these networks at the subsequent solution of various intellectual tasks.

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